Answers
Final exam in Public Finance - Spring 2017
3-hour closed book exam

By
Jakob Egholt Søgaard

Part 1: Questions on various topics

(1A) *When a municipality raises its tax on labor income it has a negative externality on the central government, when the central government also taxes the same labor income.*

True. When a municipality raises its tax rate t_g on labor income (L) any behavioral effects from the higher marginal tax rate on aggregate labor supply also reduces the tax revenue of the central government. This is a negative (fiscal) externality on the central government.

More formally we have the following: When a municipality raises its tax rate t_g on labor income (L) it affects its own tax revenue given by $R_g = L \cdot t_g$ through both a mechanical (dM) and behavioral effect (dB):

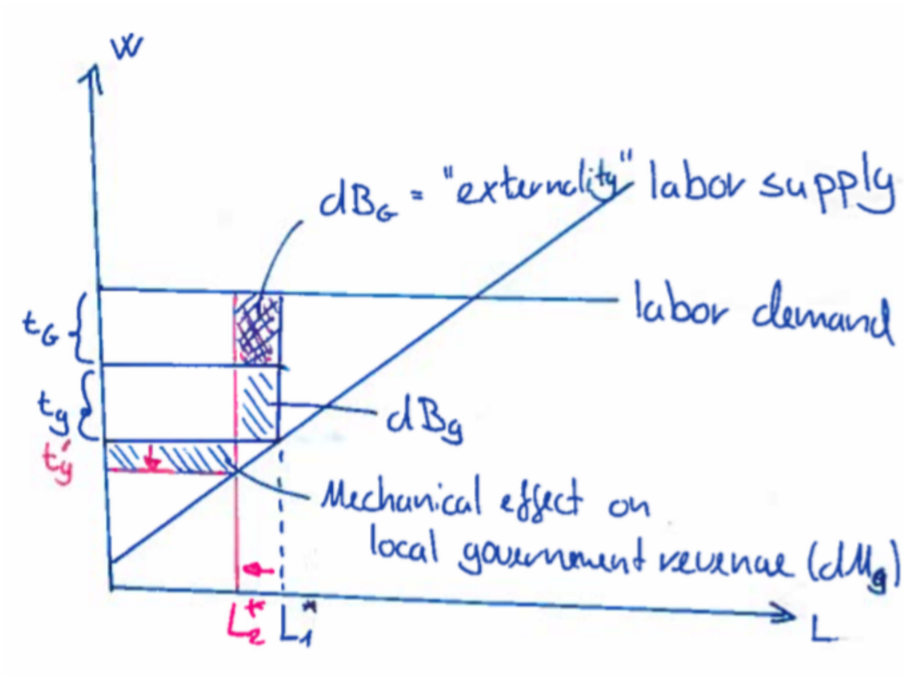
$$\frac{dR_g}{dt_g} = dM_g + dB_g = L + t_g \cdot \frac{dL}{dt_g}.$$

The municipality should take both of these effects into account when considering to raise its tax rate. However, when the central government taxes the same labor income and thereby collecting the revenue $R_G = L \cdot t_G$, we have that

$$\frac{dR_G}{dt_g} = dB_G = t_G \cdot \frac{dL}{dt_g} < 0,$$

because a higher t_g imply a lower after-tax and hence a reduction in aggregrate labor supply (assuming that the labor supply elasticity is positive). This is a negative (fiscal) externality on the central government.

The negative externality may also be illustrated as in the figur below.



It may also be mentioned that a higher t_g in one municipality might create a positive fiscal externality on other municipalities, because a higher t_g might induce some of the citizens (with the highest incomes) in the affected municipality to move to other municipalities. Thus a higher t_g might create a positive *horizontal* externality on the other municipalities.

(1B) *All else equal, a higher labor supply elasticity imply that the revenue maximizing tax rate in a linear tax system is higher.*

False. A higher labor supply elasticity implies a lower revenue maximizing tax rate in a linear tax system. To see this consider the effect of a higher tax rate (t) on the tax revenue $R = t \cdot L$, where L is total labor income:

$$\frac{dR}{dt} = dM + dB = L + t \cdot \frac{dL}{dt} = L\left(1 - \frac{t}{1-t} \cdot \frac{dL}{d(1-t)} \frac{1-t}{L}\right) = L\left(1 - \frac{t}{1-t} \cdot \epsilon\right),$$

where $\epsilon = \frac{dL}{d(1-t)} \frac{1-t}{L}$ is the labor supply elasticity. The revenue maximizing tax rate $\hat{t}$ is given by:

$$\frac{dR}{dt} = L\left(1 - \frac{t}{1-t} \cdot \epsilon\right) = 0 \Leftrightarrow \hat{t} = \frac{1}{1+\epsilon}.$$

From this it is clear that a higher ϵ imply a lower $\hat{t}$.

The relationship between the revenue maximizing tax rate $\hat{t}$ and ϵ stems from the following: a higher ϵ implies that workers respond more to a tax change, which for any given initial tax rate increases the behavioral effect of a marginal tax increase on the government revenue. At the same time, a higher ϵ does not affect the mechanical effect of an increase in the marginal tax rate. Hence the level of t where the mechanical effect equals the loss due to the behavioral effect is lower when ϵ is higher.

(1C) *In the paper "The Incidence of Mandated Maternity Benefits" by Jonathan Gruber (published in The American Economic Review in 1994), the empirical results support that employers are able to shift some of the costs of mantated benefits to the employees.*

True. The empirical analysis in Gruber (1994) shows that the extra health insurance costs of the firms due to mandated maternity benefits are shifted to the employees through lower wages. The overall conclusion is that the degree of shifting is significantly different from 0% but not significantly different from 100% (full shifting).

The empirical analysis exploits that some states passed laws prohibiting treating pregnancy different from "other illness" and thereby imposing higher health insurance costs on firms. This creates exogenous variation in labor costs of fertile women compared to other groups in the states passing the laws (within state variation) and also exogenous variation in labor costs of fertile women living in the states passing the laws compared to fertile women living in other states (across state variation). Gruber constructs a DiDiD estimator that exploits the exogenous variation in both dimensions at the same time.

This method controls for state-specific time trends such as business cycle effects (using the within-state difference) and group-specific time trends such as women entering the labor market (using the between-state difference). The identifying assumption is therefore identical trend differences between fertile women and others in treatment states and control states without the reform. This is a weaker assumption than the common trend assumption underlying the standard DiD identification strategy and is possible because of the two dimensions of variation in the data.

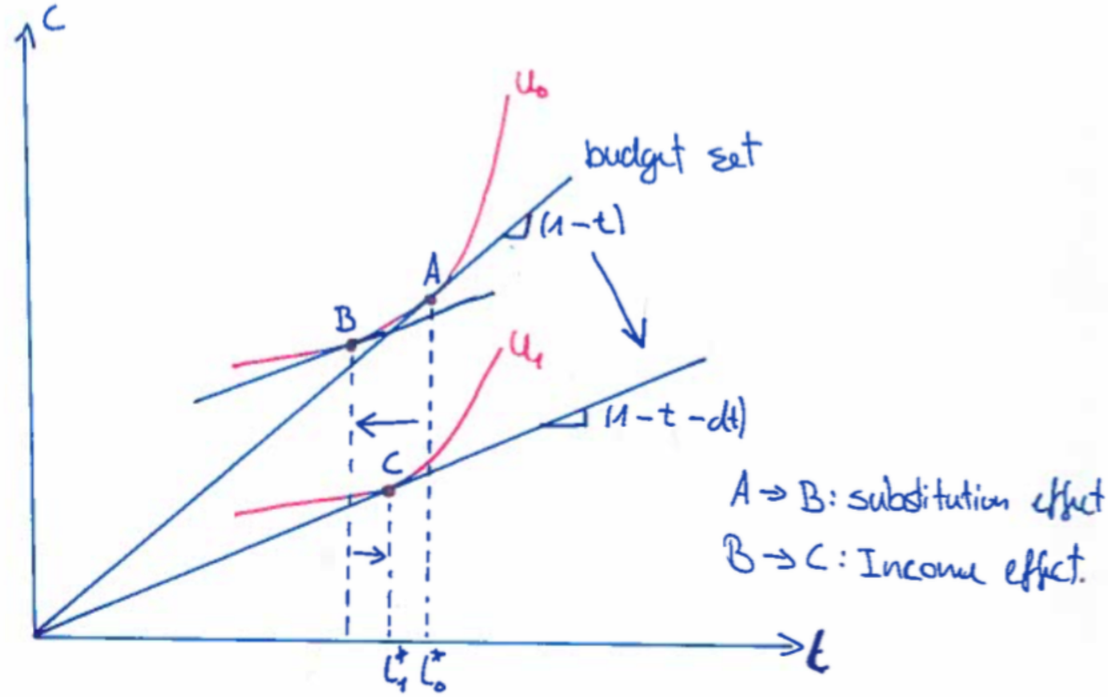
It may also be mentioned that the degree of shifting of the costs (i.e. the economic incidence) is determined by the relative size of the elasticities of labor supply (ϵ_S) and labor demand (ϵ_D). Formally we have that the share of the economic burden borne by workers can be approximated by $\frac{\epsilon_D}{\epsilon_S+\epsilon_D}$, while the share on firms can be approximated by $\frac{\epsilon_S}{\epsilon_S+\epsilon_D}$. Hence full shifting would be the case when $\epsilon_D \gg \epsilon_S$.

Part 2: Excess burden and taxes on labor income

Consider an economy where the government taxes labor income with a constant tax rate t. Individuals get utility from consumption c and disutility from supplying labor l captured by the utility function $u(c, l)$. The individuals' budget constraints are given by $c = (1 - t)l$, where the pre-tax wage rate is assumed to be fixed and normalized to 1.

(2A) *Illustrate in a diagram with l on the primary axis and c on the secondary axis the initial optimum of an individual. How does the optimum change if the tax rate is increased from t to $t + dt$? commont on the directions of the income and substitution effects, respectively.*

The initial optimun and the move to the new optimum is illustrated in the figure below.

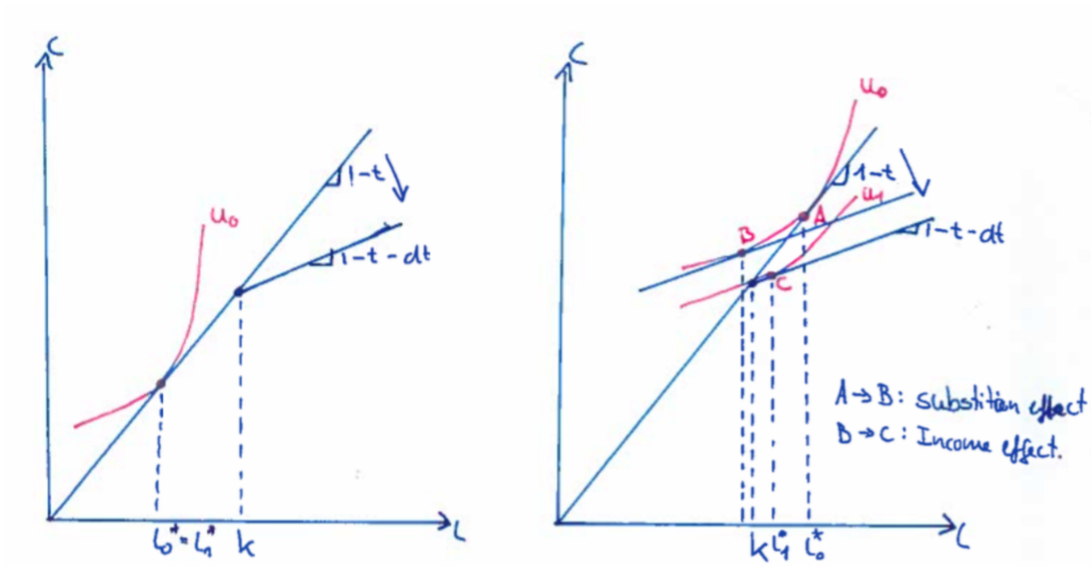


A increase in the tax rate from t *to* $t + dt$ decreases the slope of the budget, and the optimum of the individual moves from point A to point C. The substitution effect can be illustrated by changing the slope of the budget constaint, but keeping the utility level of the individual fixed (pivoting around the initial indifference curve u_0). The optimum in this situation (point B) will always lie to the left of point A implying that the substution effect decreases labor supply. The substitution effect is driven by the higher marginal tax rate which reduces the gain in consumption from working an extra hour.

The income effect is given by the move from point B to C and implies an increase in the labor supplied by the individual. The income effect is driven by the fact that the higher tax rate reduces the consumption of the individual (reducing the utility level), which induces the individual work extra.

(2B) *How does the answer to (2A) change if the increase in marginal tax rate only apply to income above a given threshold K. What can be said about the size of the income and substitution effects in this case relative to the general tax increase in question (2A)?*

The initial optimun and the move to the new optimum, given an increase in the marginal tax rate that only applies over a threshold K, is illustrated in the figures below. The figure to the left shows an individual with initial labor (income) below K, who is (of course) unaffected by the tax increase. The figure to the right shows an individual with initial labor (income) above K, who change his optimum from point A to point C.



Like in (2A) we can dekompose the change from A to C in an income and a substitution effect. The substition effect is the same as in (2A) because the change in the marginal tax rate is the same, however the income effect is smaller because the higher tax rate only apply to income above K and hence the reduction in the consumption (utility) of the individual is smaller.

Another way to see this is to consider the effect on the budget constraint in (2A) and (2B). In (2A) the budget contraint changes to $c = (1 - t - dt)l$, while it in (2B) changes to

$$c = (1 - t)min(l, K) + (1 - t - dt)max(l - K, 0),$$

which for $l > K$ implies

$$c = (1-t)K + (1-t-dt)(l-K) = dt \cdot K + (1-t-dt)l.$$

Hence the change in the budget constraint of an increase in the tax rate above K is the same as the general tax increase with the addition of the term $dt \cdot K$, which is the same as a lump sum transfer to the individual (the transfer constitute the value of the saved taxes from the fact that dt only apply above K. It is this (implicit) lump sum transfer that reduces the income effect compared to (2A).

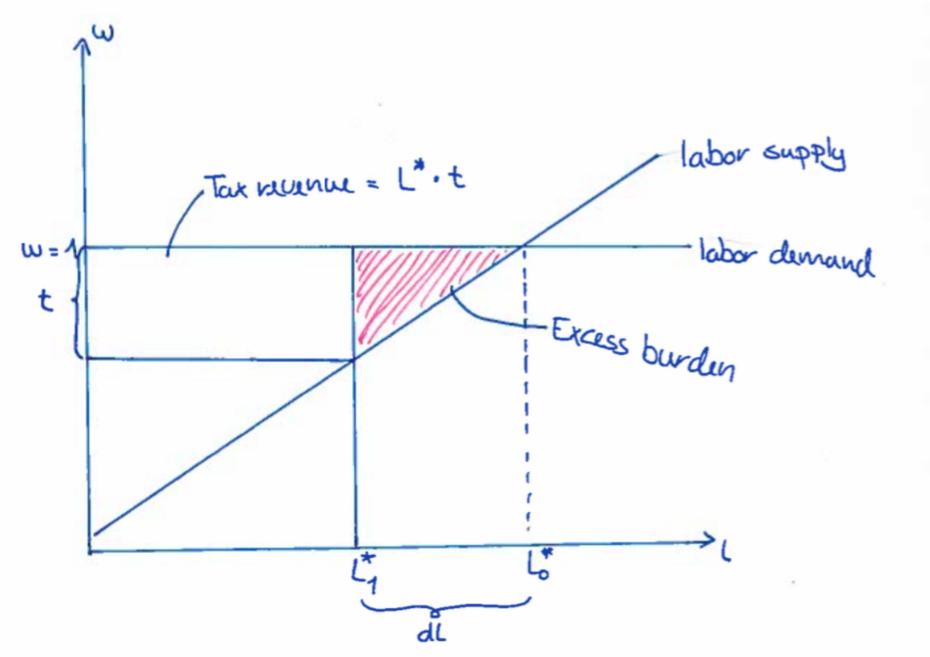
Assume that the optimization of all individuals in the economy generate an upwards sloping aggregate labor supply curve.

(2C) *Illustrate the equilibrium in the labor market with a given tax rate t in a diagram with labor supply/demand l on the primary axis and the wage rate (w) on the secondary axis. Show that the excess burden (deadweight loss) can be approximated by:*

$$EB \approx \frac{1}{2}\varepsilon \cdot t^2 \cdot l_0, \qquad \textit{where } \varepsilon = \frac{dl/l}{dw/w} = -\frac{dl/l_0}{t/1} \tag{1}$$

is the labor supply elasticity measured from the no–tax equilibrium where $l = l_0$ (recall that with $t = 0$ the after-tax wage rate (w) is equal to the pre-tax wage rate normalised to 1, which means that $dw = -t$).

The equilibrium is illustrated in the figure below.



The excess burden (or dead weight loss) can be approximated by the area of the triangle, which is given by 0.5 times the base $(l_0 - l_1)$ times the height t. The base can also be expressed as $-dl = -(l_1 - l_0)$, where dl is the change in labor supply from the introduction of the tax rate. From this we can write:

$$EB \approx -\frac{1}{2} \cdot t \cdot dl = \frac{1}{2} \varepsilon \cdot t^2 \cdot l_0,$$

where the second equality sign follows from the definition of the labor supply elasticity $\varepsilon = -\frac{dl/l_0}{t/1} \Leftrightarrow -dl = t\varepsilon l_0$.

(2D) *Assume that ε is constant and derive the marginal excess burden (MEB) of a small tax increase. How do ε and t affect the size of MEB and what is the intuition for these effects?*

Differentiating (1) wrt. t gives the following:

$$MEB = \frac{dEB}{dt} \approx \varepsilon \cdot t \cdot l_0.$$

The MEB is linearly increasing in both $t$and ϵ. The intuition for this is the following:

- t: In equilibrium individuals have optimized so that the disutility from the last unit of labor supplied is equal to gain in terms of extra consumption. This implies that individuals are indifferent about supplying one unit of labor more or less. However with an pre-exiting tax t, the wage that individuals require to work an extra hour is lower than that the employers are willing to pay and this "social surplus" is exactly equal to the tax rate. When a small increase in t decrease l by one unit, the loss in term of social surplus is therefore equal to t.

- ε: A higher labor supply elasticity implies that a given tax increase, reduces the labor supplied more. And a larger reduction in labor supply implies a larger reduction in social surpluss.

The answer may also be assisted by an illustration of the effect of an increase in t in a figure like the one in 2C.

(2E) *Explain why we should only include behavoiral responses due to the substitution effect when computing the excess burden? That is: why should we use the compensated elasticity in (1) and not the uncompensated? Illustrate how the diagram from the answer to (2C) would change if you had drawn the compensated labor supply curve instead of the uncompensated.*

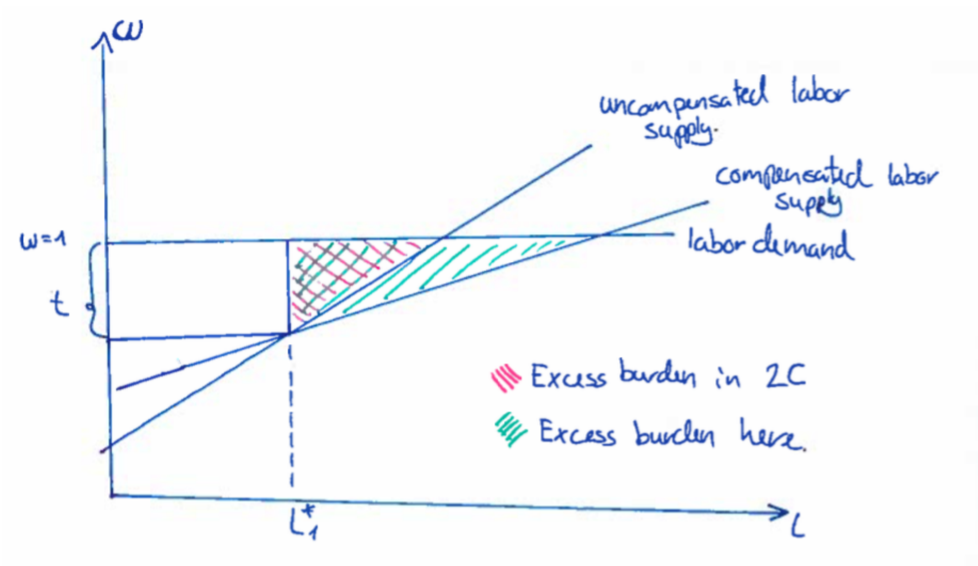
The key question in measuring the excess burden is to what extent a tax affect Pareto efficiency. One way to answer this is to ask: How much more tax revenue could the government collect from an individual without making him worse off?

This can be answered using the equivalent variation which measures the size of the lump sum tax that gives the individual the same utility as with the tax on labor income. The difference between the lump sum tax and the revenue from the tax on labor income therefore equals the excess burden. Because a lump sum tax also creates income effects these could not be included when computing the excess burden.

The labor supply curve we used in question (2C) was uncompesated and therefore driven by two effects:

- Higher taxes reduce the gain from workning an extra hour $\rightarrow$ individuals work less even though firms are willing to pay more than the individuals require to work an extra hour (substitution effect).
- Higher taxes reduce the individuals' after tax income/consumption $\rightarrow$ individuals work more to earn extra (income effect).

As the income effect works in the opposite direction than the substitution effect, the compensated labor supply curve is flatter/more elastic than the uncompensated and hence the excess burden is larger in this case as illustrated in the figure.



Part 3: Extensive labor supply responses

Consider a model where individuals have to decide whether to enter the labor market or not. If they enter the labor market they work a fixed number of hours ($\bar{h}$) at a fixed wage rate (w). Working thereby gives them fixed earnings of $Y = w\bar{h}$, however at the same time the individual has to pay a tax T. Working is furthermore associated with a fixed disutility cost q. Individuals who do not work receive benefits B. Utility is defined as:

$$u = \begin{cases} Y - T - q & \textit{if working} \\ B & \textit{if not working} \end{cases} \tag{2}$$

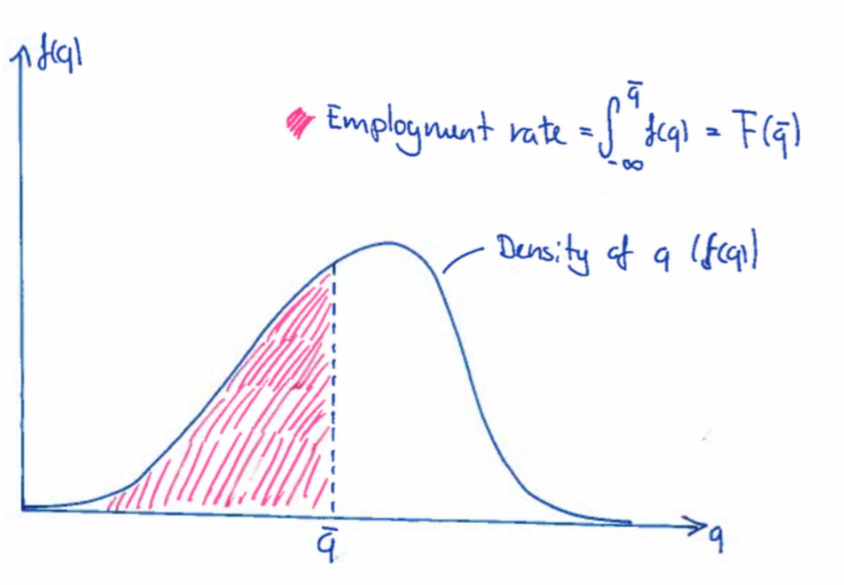
The fixed cost, q, is distributed heterogeneously across individuals according to a density function $f(q)$ with cumulative distribution function $F(q)$.

(3A) *Derive the fixed cost of the marginal individual just willing to work, call it $\bar{q}$. Draw a diagram with q along the primary axis and $f(q)$ along the secondary axis. What is the employment rate (E) in the diagram?*

The marginal individual, who is indifferent between working or not is charaterized by:

$$Y - T - q = B \Leftrightarrow Y - T - B = q \equiv \bar{q}$$

All individuals with a $q < \bar{q}$ choose to work, which gives the employment rate of $E = F(\bar{q})$. This can be illustrated as in the figure below.



It can be noted that the employment rate depends negative on both T and B, as both reduce the income difference between employment and unemployment. $T + B$ is often called the participation tax.

The government net revenue is given by $R = E \cdot T - (1 - E) \cdot B$.

(3B) *Show that the effect of change in* T *on the government net revenue can be written as:*

$$\frac{dR}{dT} = E\left[1 - \theta\frac{a+b}{1-a-b}\right], \tag{3}$$

where $a = T/Y$, $b = B/Y$ *and* $\theta = \frac{dE/E}{d(Y-T-B)/(Y-T-B)}$ *is the elasticity of the employment rate wrt.* $Y - T - B$. *Give an intepretation of the equation (3).*

Rewrite the government net revenue and diffentiate wrt. T gives:

$$R = E \cdot T - (1 - E) \cdot B = E \cdot (T + B) - B$$

$$\frac{dR}{dT} = E + \frac{dE}{dT} \cdot (T + B) = E\left(1 - \frac{dE}{d(Y-T-B)}\frac{1}{E} \cdot (T+B)\right)$$

$$= E\left(1 - \frac{dE}{d(Y-T-B)}\frac{Y-T-B}{E} \cdot \frac{T+B}{Y-T-B}\right) = E\left(1 - \theta\frac{T+B}{Y-T-B}\right)$$

$$= E\left(1 - \theta\frac{a+b}{1-a-b}\right).$$

This equation captures the two effects on the government net revenue from an increase in T. First the mechanical effect, which is given by the extra tax revenue from the individuals who are initially employed. Second the behavoiral effect, which is given by the loss in net revenue from the individuals, who decide to leave the labor markedet, when the tax is increased. The loss in net revenue from each one leaving the labor market is given by a (lost tax revenue) + b (extra benefits payments).

In the paper "Labor Supply Response to the Earned Income Tax Credit" by Eissa and Liebman (published in the Quarterly Journal of Economics in 1996), the authors investigate the effect of the 1986 expansion of the earned income tax credit (EITC) for single women with children on their labor force participation. The EITC (in Danish: "beskæftigelsesfradrag") offers a tax credit for eligible individuals contingent on earning positive income and having a qualifying child. Below is a copy of Figure IV from the article, showing the structure of the EITC pre and post reform.

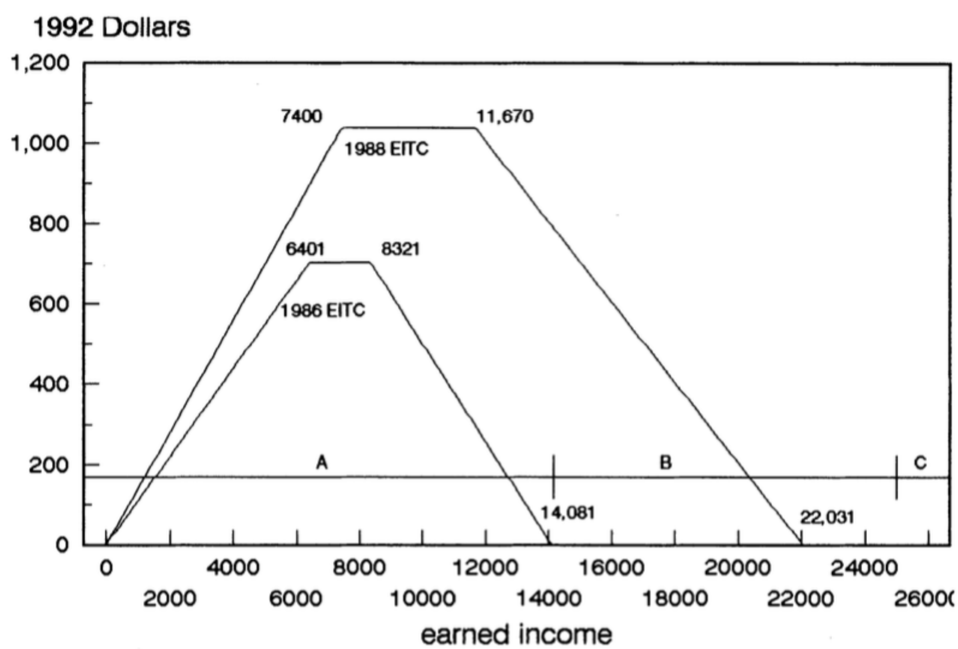


Figure IV
1986 and 1988 Earned Income Tax Credit

(3C) *Explain how an EITC may affect labor force participation in the context of the model* *presented above. How do you expect the reform to affect labor force participation of the targeted group?*

There are two questions here. First, an EITC lowers the tax burden on low-income earners on the labor market without increasing disposable income for those outside the labor market. That is, T decreases and B is unchanged. This amounts to a drop in the participation tax rate $(a + b)$, and according to our model, this should increase employment relative to a situation without an EITC.

Second, the TRA86 expansion of the EITC amounts to a further decrease in the size of the participation tax. In the model, this leads to an increase in the employment rate for the affected group, single women with a qualifying child. It may be noted that the actual reform affects marginal tax rates differently in different income regions as well as the participation tax rate, which complicates matters. But given our simple model without intensive margin optimization, we would expect labor force participation of the targeted group to increase.

Eissa and Liebman (1996) use the reform to estimate the impact of the EITC expansion on labor force participation of single women with children. Below is a copy of Table II from the article showing their main estimate.

TABLE II
LABOR FORCE PARTICIPATION RATES OF UNMARRIED WOMEN

	Pre-TRA86 (1)	Post-TRA86 (2)	Difference (3)	Difference-in-differences (4)
A. *Treatment group:*				
With children [20,810]	0.729 (0.004)	0.753 (0.004)	0.024 (0.006)	
Control group:				
Without children [46,287]	0.952 (0.001)	0.952 (0.001)	0.000 (0.002)	*0.024 (0.006)*

(3D) *Describe the empirical analysis and explain, using Table II above, how the authors arrive at their estimate. Does their estimate have the expected sign, given your argumentation in (3C)? What is the main identifying assumption needed for the estimate to be the causal effect of the EITC on the labor supply of single women with children?*

Eissa and Liebman (1996) estimate the impact of the EITC expansion using a difference-in-differences (DiD) estimation framework. In order to use this estimator, they need two similar groups, where one is affected by the reform and one is not. The affected group (treatment group) is single women with a qualifying child, and the unaffected group (control group) is single women without children.

In Table II we see the simple DiD estimation table without controlling for observable characteristics of the two groups. Denote by E_t^i the labor force participation of group i (= **T**reatment or **C**ontrol) at time t (**pre** or **post** reform). Column 1 shows pre-reform labor force participation of the treatment group (top row) and the control group (bottom row). Column 3 shows the time differences of each group

$$E_{post}^T - E_{pre}^T = 0.753 - 0.729 = 0.024$$

$$E_{post}^C - E_{pre}^C = 0.952 - 0.952 = 0.000.$$

Column 4 shows the DiD estimate as

$$\left(E_{post}^T - E_{pre}^T\right) - \left(E_{post}^C - E_{pre}^C\right) = 0.024 - 0.000 = 0.024$$

The estimate of the impact of the reform has the expected sign as the expansion of the EITC seems to have increased the labor force participation of the target group.

However the DiD estimator only gives the causal effect of the EITC if the common/parallel trend assumption is fullfilled. The common trend assumption states that, in the absence of the reform, the change in the employment rate of the two groups should be the same.

It is also possible to show this using the Rubin Causal Model (RCM). The observed change in the participation rate of the treatment group can be written as

$$\begin{aligned} dE^T &= E(T=1, q_{post}^T) - E(T=0, q_{pre}^T) \\ &= \underbrace{E(T=1, q_{post}^T) - E(T=0, q_{post}^T)}_{\text{Treatment effect}} + \underbrace{E(T=0, q_{post}^T) - E(T=0, q_{pre}^T)}_{\text{Bias term}}, \end{aligned}$$

where T is an indicator for whether the group is treated or not and q_t^i is other factors affecting the employment level of group i at time t. The identifying assumption in the DiD estimator is that bias term above is equal to the observed change in the employment rate of the control group. That is:

$$\underbrace{E(T=0, q_{post}^T) - E(T=0, q_{pre}^T)}_{\text{Bias term}} = dE^C = E(T=0, q_{post}^C) - E(T=0, q_{pre}^C)$$

Given this assumption the DiD estimate is

$$dE^T - dE^C = \underbrace{E(T=1, q_{post}^T) - E(T=0, q_{post}^T)}_{\text{Treatment effect}}$$

(3E) *Describe how you could validate the main identifying assumptions needed in (3D) and what kind of data you would need to do so.*

The key assumption in the DiD estimator is the common trend and one way to validate if this assumption seems plausible is to consider the evolution of the employment rates of the treatment and control group before and/or after the reform. If the employment rates move in parallel in these years it speaks to the validity of the common trend assumption.

More formally this can be tested by running the same DiD estimators in non-reform years. The estimates from these "Placebo tests" should be insignificant.